Lab Manual
Fluid Dynamics Practical

January 1999, Ir. J.C. van Muiswinkel

MEAH-192

Revised in December 1999 by
Ir. C. Poelma
Ir. P. Bernardini

Delft University of Technology
Laboratory for Aero- & Hydrodynamics
Rotterdamseweg 145
2628 AL Delft

email: j.c.vanmuiswinkel@wbmt.tudelft.nl
c.poelma@wbmt.tudelft.nl
p.bernardini@wbmt.tudelft.nl

Contents:

MOTIVATION:

For students with little or no experience in the standard measuring techniques used in fluid dynamics, three experimental set ups are presented here which can be used to gain some first experience. In this section, a motivation for the choice of these particular experiments is given.

At “normal” temperature and pressure, gasses and liquids can be assumed to be homogeneous and continuous; i.e. the smallest volumes that we consider in the experiments, are still composed of a very large number of molecules. At rest, the state of a fluid is determined by the scalar properties p, r and T (pressure, density and temperature). In addition to these three properties, the three components of the local velocity vector $\boldsymbol{u}$ are required for a flowing fluid. The most practical way of describing a flow is the one in which the local values of p, r, T and $\boldsymbol{u}$ are given as so-called field properties in a fixed frame of reference. At each position these field properties are a function of time in the case of a non-stationary flow. Six equations are therefore needed to describe the flow field: three scalar equations and a vector equation. These are:

- the equation of continuity;
- the energy equation;
- the equation of state;
- the three momentum equations.

The equation of continuity represents conservation of mass, the energy equation represents conservation of the different forms of energy and for a non-flowing medium this is generally known as the first law of thermodynamics. The momentum equation represents the conservation of momentum also known as the second law of Newton ($\boldsymbol{F} = m \cdot \boldsymbol{a}$). The equation of state can be formulated as f(p,r,T) = 0 and for an ideal gas it is given by p =RrT.

Because of the deformation of fluid elements in a moving fluid, additional stresses occur which have to be taken into account in the equations. For a so-called Newtonian fluid the relationship between the deformation and the resulting stress is given by a linear function. Most gasses and liquids behave like Newtonian fluids. Exceptions are: colloidal solutions, concentrated solutions of macromolecules, emulsion, suspensions and the “Bingham” liquids. These exceptions are beyond the scope of this manual.

For a one-dimensional, unidirectional flow, i.e. a flow with one velocity component say u, which is a function of one space co-ordinate say y, one can write for the tangential or shear stress:

$$t = m \frac{du}{dy} \qquad (1)$$

where m is the dynamic viscosity of the fluid, which is a material property.

For a three-dimensional flow, the corresponding relationship is composed of similar terms, but it will be obviously more complex. Apart from the aforementioned tangential stress, also a viscous normal stress can occur. In general, this can be neglected, since in many flows the longitudinal or streamwise velocity gradients are much smaller than the transversal ones.

In the most general case of a flowing, compressible fluid, the solution of the governing equations is very complicated. This is caused by the fact that the momentum equations are non-linear, making the superposition of solutions impossible. A simplification is possible if the fluid can be regarded as incompressible. If the fluid is moreover homogeneous, the equation of state reduces to $r = constant$ and the first law of thermodynamics,

$$dq = de + p\,d\left(\frac{1}{r}\right) \qquad (2)$$

will simplify to:

$$dq = de \qquad (3)$$

This means that the heat (q) added to the fluid will only cause an increase of internal energy (e). Because the density r is constant and thus independent of the temperature and pressure, the pressure p and the velocity $\boldsymbol{u}$ follow from the solution of the equation of continuity and the momentum equation. With p and $\boldsymbol{u}$ known, the temperature can be calculated from the energy equation. Nevertheless the exact solution of these equations is still impossible because the non-linearity to which we have referred above, remains.

It is obvious that liquids can be treated as incompressible fluids. However, this assumption also holds for gasses at low velocities, since the density differences – due to pressure differences – are very small. A simple calculation shows that in air, at a speed of 100 [m/s], the error in an incompressible calculation of the dynamic pressure, $1/2r\,u^2$, is in the order of a few percent. Therefore, at low speeds air can be considered as incompressible.

The pressure forces on a fluid particle carrying out an incompressible stationary flow must equal the inertial forces and the shear stresses due to viscosity on the particle. The inertial forces are caused by the fact that the particle does not have a constant velocity when it moves through the flow field. A fluid particle must accelerate if it moves from a region of low flow speed to a area of high flow speed and vice versa. The ratio of inertia forces to viscosity forces in a flow is expressed by the Reynolds number, which is given by:

$$Re = \frac{inertial\ forces}{viscous\ forces} = \frac{u^{*}.\dfrac{u^{*}}{l^{*}}}{n.\dfrac{u^{*}}{l^{*2}}} = \frac{u^{*}.l^{*}}{n} \qquad (4)$$

This dimensionless number also appears when we write the equations of motion for an incompressible flow (the Navier-Stokes equations) in their dimensionless form.

In this definition, u^* en l^* are a characteristic velocity and length scale for a given flow configuration. For a circular pipe, for instance, l^* equals the diameter D of the pipe and u^* the mean flow velocity (which is given by the volumetric flow rate divided by the cross section area). On the other hand for flow around a body, u^* is the undisturbed velocity and l^* the length or width of the object.

For most flows in industrial applications, $Re >> 1$, i.e. the inertial forces in the flow are dominant with respect to the viscous forces. This does not mean that the influence of viscosity can be neglected all together. Instead $Re >> 1$ implies that the influence of the viscous shear stress is limited to a very thin layer near a wall: the so-called boundary layer. Here, the velocity increases from the no-slip boundary condition at the wall to some finite value, characteristic of the inviscid outer flow, at the edge of the boundary layer. Apart from the adjustment to the boundary condition at the wall, the boundary layer can be also considered as the flow region where the mechanical work by the shear stress is completely converted into heat. The "high grade" mechanical energy, i.e. kinetic energy and displacement work of the pressure forces, is degraded to "low grade" heat energy. This process is also referred to as dissipation and it constitutes a "loss" of mechanical energy. Another example of an area dominated by the effects of viscosity is the shear layer which surrounds a free jet and in which the flow adjusts from the velocity inside the jet to the velocity field in the region outside the jet.

Let us continue with the boundary layer near a solid wall. Consider an increasing velocity, i.e. an increasing Reynolds number. At first, i.e. for low velocity, the fluid layers in the boundary layer glide smoothly over each other. This case is referred to as the "laminar boundary layer" and the velocity at each position is constant as a function of time. When the Reynolds number is increased above a certain critical value, the velocity profile becomes unstable leading to a turbulent boundary layer. This is called transition and it is one of the most fundamental problems in fluid dynamics. In a turbulent boundary layer the velocity is no longer constant as a function of time but consists of a time-averaged or mean velocity on which are superposed random velocity fluctuations. Near the wall these fluctuations are in the order of 10% of the mean velocity. Because of these fluctuations, the shear stresses become much larger and the velocity profile will change significantly from the laminar case. In most engineering problems, *Re* usually lies above the critical value, so that turbulent boundary layers are the most common.

As mentioned earlier, the flow of an incompressible medium is determined by the momentum equations and by the equation of continuity. For some applications, these equations are needed in their integral form. The integration of the momentum equations can be done in two ways:

1. By integrating the momentum equations over a volume we obtain an integral form which gives the conservation of momentum in a finite volume. The surface surrounding this volume is denoted as a control surface. For a stationary flow, it is possible to convert all volume integrals into surface integrals over this control surface. The practical importance of this form is evident because the whole flow inside the volume can be expressed solely in integrals to be evaluated at the control surface. It is often possible to choose the control surface in such a way that these integrals can be computed. Obviously, this integral form does not yield details about the flow inside the control surface, but for most technical problems, this is of minor importance.

2. In case of a stationary flow we can integrate the momentum equations along a streamline. The result can be interpreted in terms of the properties of a fluid element, which travels along the streamline. This fluid element is chosen to be either a unit of mass or a unit of volume. For the case of an inviscid fluid the integral of the momentum equation leads to.

$$p + \frac{1}{2} r(u^2 + v^2) = constant \tag{5}$$

which is know as the law of Bernoulli. In this equation, valid for a two-dimensional flow configuration, *u* and *v* are the velocity components in the x- and y-direction. The equation of Bernoulli is the equivalent of the mechanical law, which states that the sum of potential and kinetic energy is constant. Alternatively, the two terms in the Bernoulli law can be interpreted as the energy-state of the flow at a given point and the Bernoulli law states that this energy state is constant along a streamline. Let us consider a stream tube, of which the walls are made up of streamlines. We multiply the energy state at each point in the cross-section of the stream tube by the local velocity and integrate across this cross-section. The result is the (mechanical) energy transport through the cross section of the stream tube. The law of Bernoulli then states that this energy transport is the same at each cross-section of the stream tube.

Apart from the momentum equations in their integral form, the Bernoulli law is the most commonly used relationship in engineering fluid dynamics. Without these two, in essence quite simple, standard equations, engineering fluid dynamics would be unthinkable; they can be treated as a minimum basis for fluid dynamics. Provided that they are applied well, they produce many useful solutions to engineering problems. There are, however, a number of limitations to the use of these equations, especially regarding the Bernoulli law, for instance at low Reynolds numbers, when shear stresses can not be neglected. The Reynolds number therefore has to be sufficiently large so that except for boundary layers the influence of viscosity can be neglected.

After this introduction, it will be clear that there are three important examples of fluid mechanical principles that would be suitable for our introductory experiments: the laminar-turbulent transition, the integral momentum equation and Bernoulli's law. To illustrate the background and the applicability of these principles, three experiments have been selected. The focus will be on the basic physical concepts rather than on mechanical engineering aspects.

Let us give a short introduction into these experiments.

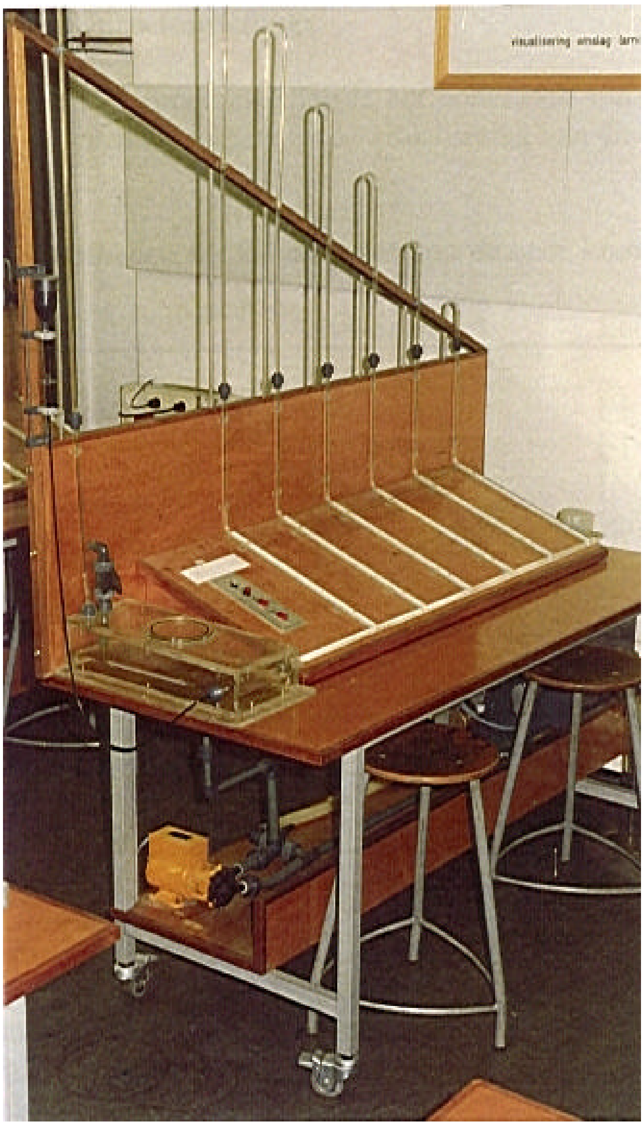
- In the experiment on laminar-turbulent transition, the set-up, a narrow tube with a diameter of 6 mm, is chosen such to ensure that both laminar and turbulent flow can be established. The characteristic differences between these two flow types, e.g. the difference in pressure drop, can be observed, as well as the influence of temperature on the viscosity (and thus on the Reynolds number). The results can be put in a so-called Moody diagram, which is used in practice. Finally, the complicated process of laminar-turbulent transition is made visible by injecting a fine jet of ink.

- The consequence of the momentum equation in its integral form can be demonstrated most simply in the classical configuration of an abrupt expansion. Here, extra attention is given to the implicit assumption that the pressure in the plane of the expansion is constant, since this assumption is not self-evident. In addition the mechanical energy flow is calculated and it is shown that part of the energy is degraded into thermal energy. Engineering examples of this geometry include the sudden expansion in diameter in a pipe, a pipe in a tank, a valve, etc.

- A second, more advanced application of the integral momentum equation is the conical convergence. To this geometry, the Bernoulli law can be also applied. By calculation, it has to be shown that this relationship is in agreement with the results of the momentum equation.

The exit jets of two last experimental set-up allow more advanced experiments. This is in particular the case for the conical convergence for which the exit jet has a flat profile over more than 75% of the diameter.

Delft, January 2000.



Experimental Set Up 1

Chapter 1
Laminar-turbulent transition in a pipe flow

1.1 Introduction

The subject of this experiment is the investigation and visualisation of the transition process from laminar to turbulent flow in a cylindrical pipe flow.

Aim of the experiment:

- Application of the double-logarithmic Moody diagram for both laminar and turbulent pipe flow
- Study of the influence of temperature on the viscosity and thus on the Reynolds number.
- Visualisation of the transition process by injecting a fine jet of ink into the flow.

1.2 Theory

Quantity	Symbol	Units
Reynolds number	Re	[-]
Friction factor	f	[-]
Pressure drop	Dp	[N/m^2]
Pipe Diameter	D	[m]
Pipe Length	L	[m]
Density	r	[kg/m^3]
Velocity	v	[m/s]
Kinematic viscosity	n	[m^2/s]
Volumetric flow rate	F_v	[m^3/s]
Wall roughness	e	[m]
Relative wall roughness	e/D	[-]

Table 1.1: relevant properties

a. Laminar flow:

For Re_D < approx. 2300, the flow is laminar and can therefore be described by the Poiseuille formula. In this case, the friction factor is given by:

$$f = \frac{64}{Re_D} \tag{1.1}$$

In this formula, Re_D is the Reynolds number based on the inner diameter D of the pipe. In a double-logarithmic Moody diagram (Moody, 1944), relationship (1.1) is represented by a straight line (see figure 1.1).

b. Turbulent flow:

For Re_D > approx. 4000, the flow is turbulent. In this case, the friction factor f depends on both Re_D and e/D. In the turbulent region of the Moody diagram, the measured values for our experiment will be near the "smooth pipe" line.

c. Transition region:

For $2300 < Re_D < 4000$, the friction factor f can not be predicted. Its value lies somewhere in the shaded area, depending on the inflow conditions and the roughness of the pipe wall. In this region, the transition process can be visualised by injecting a fine jet of ink.

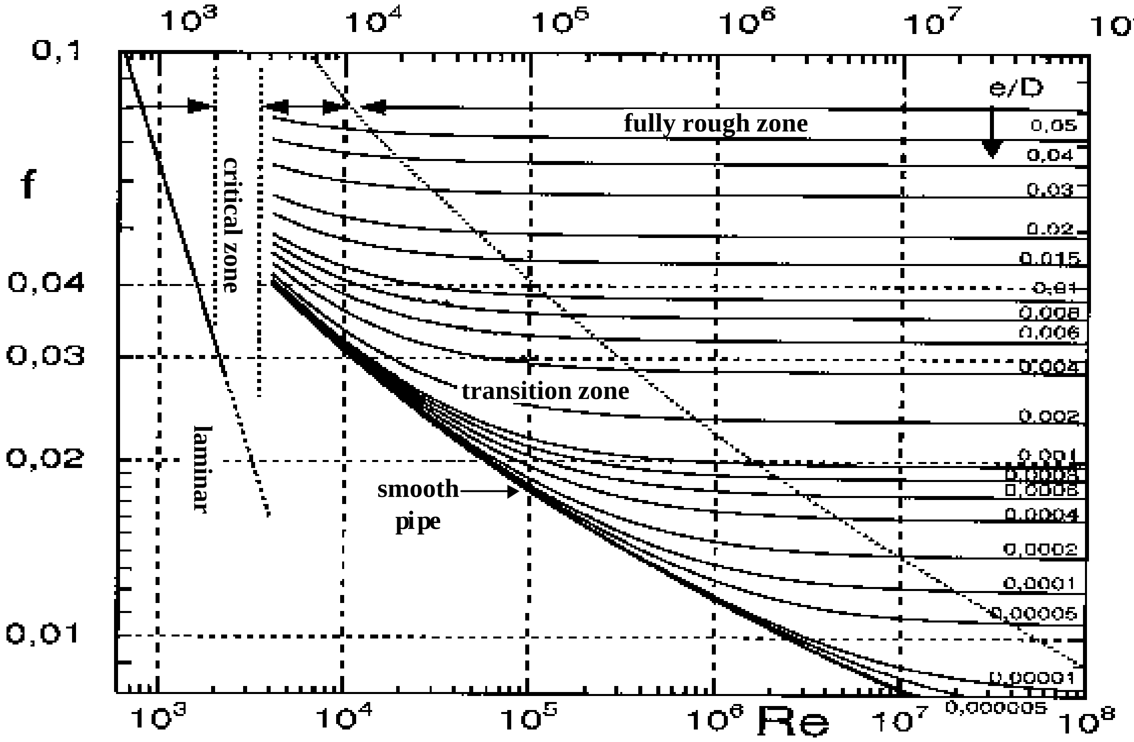


Figure 1.1: the friction factor for fully developed flow
in pipes with circular cross section

1.3 Experimental Set up

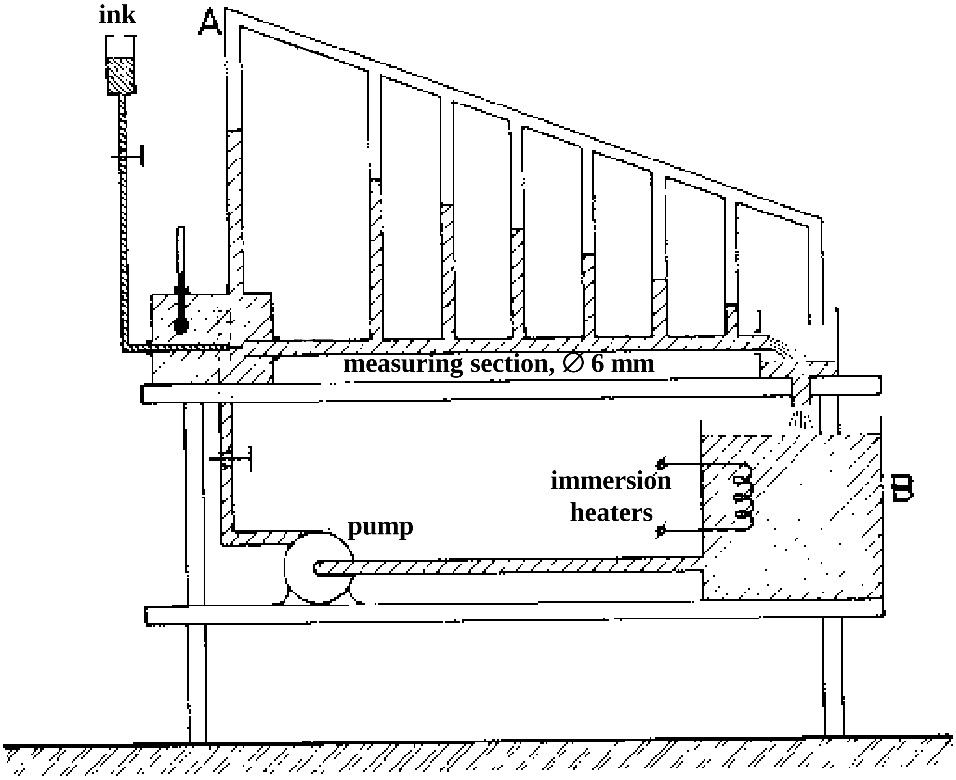


Figure 1.2: the experimental set up

The main part of the set up is a 1-meter long, 6-mm wide tube, through which water is pumped. At intervals of 20 cm, the pressure can be determined by the height of a water column in the tubes **A**. The water is collected in tank **B** and recirculated. The flow rate can be determined by a valve (V), positioned just over the transparent box at the entrance of the test section. In this box, both a thermometer and a ink injection needle are present.

In tank **B**, a float ensures a sufficient water level. The water is heated by a set of immersion heaters: a 2000 Watt heater that switches off at 54 °C and a 1000 Watt heater that switches off at 56 °. The float automatically switches off the heaters if the water level drops to low value. From tank **B**, it is possible to redirect the water into a measuring cylinder, so the flow rate can be determined accurately.

The experiment consists of measuring the pressure drop over the pipe for different flow rates and at different temperatures (56 °C and room temperature).

By adjusting the flow rate, conditions in both the laminar as well as the turbulent region can be selected. At 56 °C, almost all the measured points will be in the turbulent region; for room temperature, it is also possible to measure in the laminar region.

1.4 Carrying out the Experiment

Ensure that there is enough water in tank **B** and switch on the heating. Water can be added to the tank by opening the tap, if necessary. To reach a temperature of 56 °C takes approximately 45 minutes.

Do at 56 °C experiments with 6 different flow rates: begin by setting the pressure in tube 1 (which is a measure for the flow rate) to 75 cm H_2O. For the next experiments, lower it to 60, 45, 30, 15 and 3-4 cm H_2O. At each of these settings, measure the following quantities:

1. the pressure drop *Dp*

The pressure drop can be determined by reading out the static pressures at the six tubes. To determine the pressure drop over the 1-meter pipe, only the first and the last values are required. The values in between can be used to get a more accurate result (e.g. by least squares line fitting). The scaling on the inclined section of the panel has already been corrected: the scaling is "stretched".

2. the volumetric flow rate F_V

Attached to the test tube is a flexible hose, which can be placed into a tube that leaves the tank. The water flowing out of this tube can be collected in a measuring cylinder. For an adequate accuracy, make sure to collect at least 0.5 litres (e.g. 10-15 seconds for higher pressures and 40-45 seconds for lower pressure).

3. the kinematic viscosity *n*

Although the temperature is kept constant by means of the 1000 W immersion heater, it may vary slightly over time. The temperature can be read using the thermometer in the transparent box. The viscosity can then be read from figure 1.3.

After doing the experiments at 56 °C, the set up can be emptied and filled with fresh tap water to repeat the experiments at room temperature.

At the end of the experiment, ink can be injected to visualise the transition. Before doing this, check if the ink level is sufficient and that the injection system is not clogged. If it is not possible to unclog the tubes, ask assistance from the workshop.

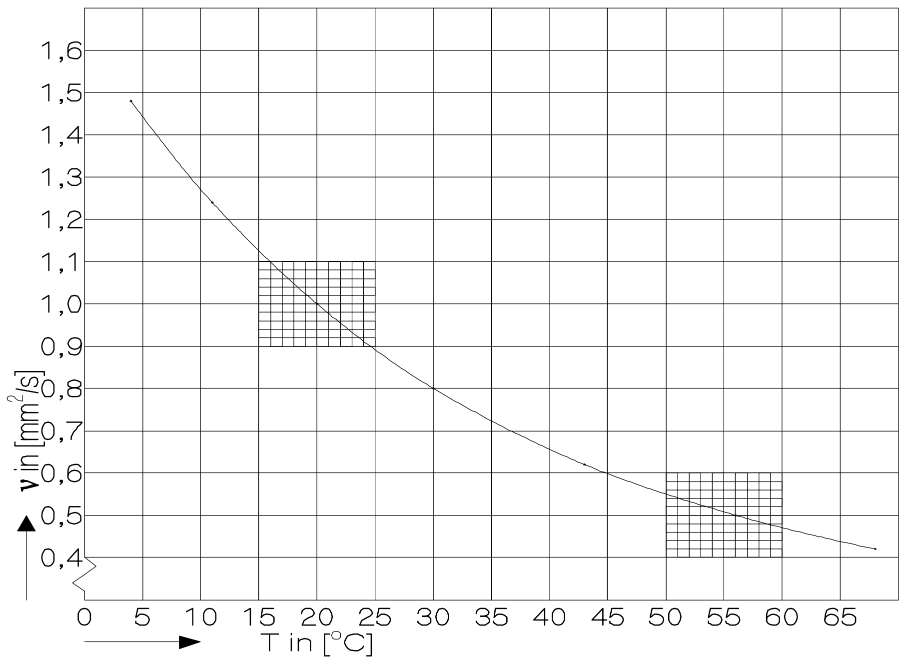


Figure 1.3: the kinematic viscosity *n* in [mm²/s] as
a function of temperature *T* in [°C]

1.5 Moody Diagram

Check the results obtained by plotting them in a Moody diagram. For this the Reynolds number and the friction factor should be determined. The Reynolds number for each of the experiments can be calculated from the measured values of the flow rate and of the viscosity:

$$Re_D = \frac{vD}{n} \qquad (1.2)$$

The velocity in equation (1.2) can be calculated from the flow rate with help of the pipe diameter:

$$\Phi_v = \frac{1}{4} p\ D^2 . v \qquad (1.3)$$

Solving for *v* and substituting the result in (1.2) gives:

$$Re = \frac{4\Phi_v}{pnD} = C_1 . \frac{\Phi_v}{n} \qquad (1.4)$$

The constant C_1 in (1.4) is determined by the pipe diameter. This diameter is not exactly 6 mm and is therefore determined by calibration. The value for C_1 is written on the set up and is only valid when the flow rate is given in ml/s and the viscosity in mm²/s.

The friction factor can be calculated from the pressure drop. For a fixed pipe length L, the pressure drop is given by:

$$\Delta p \;=\; f.\frac{L}{D}.\frac{1}{2}\, r\, v^2 \qquad (1.5)$$

Solving for f and substituting (1.3) for v^2 gives:

$$f \;=\; \frac{p^{2} D^{5}}{8\, rL}.\frac{\Delta p}{\Phi_v^2} \;=\; C_2.\frac{\Delta p}{\Phi_v^2} \qquad (1.6)$$

The value of constant C_2 is dependent on D even more than C_1 ($\sim D^5$) and is written on the set up. The pressure drop and the flow rate are respectively given in cm H_2O and ml/s.

With equations (1.4) and (1.6) the Reynolds numbers and friction factors can be calculated for each of the experiments. Draw the results in a Moody diagram.

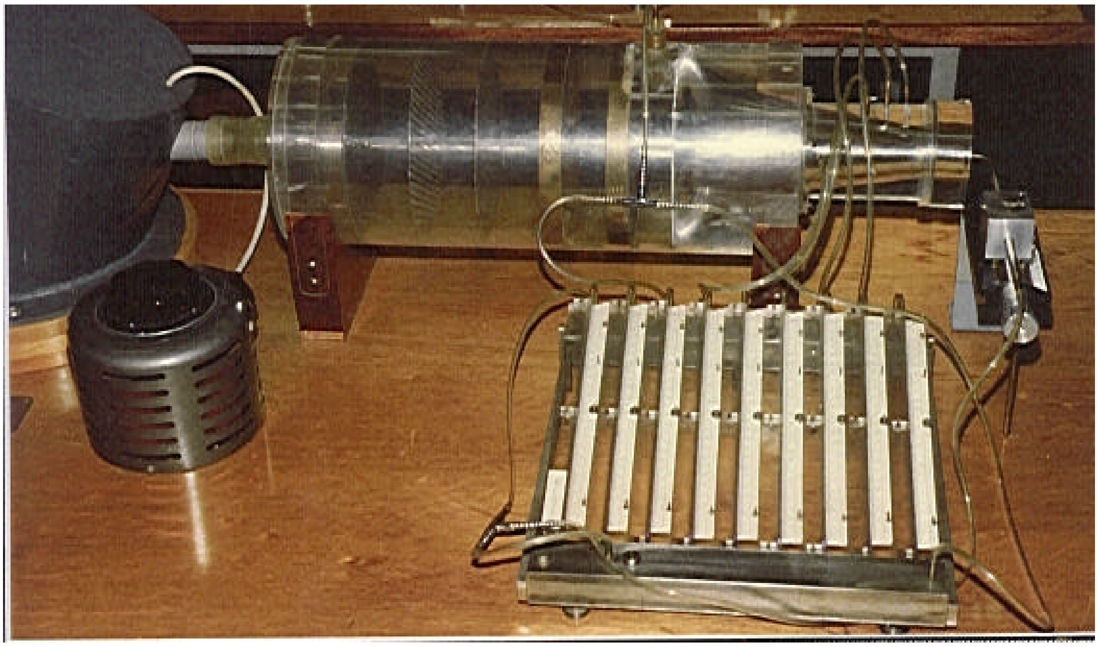
To get an idea of the reproducibility, three extra measurements can be done. Choose settings that gave the least satisfying results, i.e. the values that were not close to the “smooth pipe” curve.

Questions

a) Why is it difficult or even impossible to demonstrate the transition at high temperature?
b) Why is it difficult to determine the value for *Dp* when the flow is in the transition region?

References

[1.] Moody, L.F., “Friction Factors for Pipe Flow”, *Transactions of the ASME*, **66**, 8, November 1944, pp. 671-684.



Experimental Set Up 2

Chapter 2

Flow in a conical convergence

2.1 Introduction

In this set up we investigate the flow in a converging tube. The aim is:
- Check the law of Bernoulli by means of measurement and calculation.
- Check the momentum balance by means of measurement and calculation.

2.2 Theory

Quantity	Symbol	Units
Settling chamber pressure	p_0	[N/m^2]
Static pressure	p	[N/m^2]
Velocity	v	[m/s]
Diameter	d	[m]
Radius	r	[m]
Density	r	[kg/m^3]
Force	F	[N]

Table 2.1: quantities involved

Bernoulli law:

Air flows from a big settling chamber into a converging tube. The pressure in the settling chamber is p_0, measured with respect to the atmosphere. The velocity in the settling chamber is taken to be negligible.

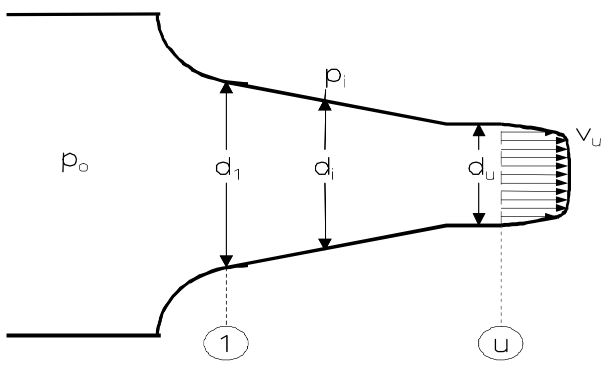


Figure 2.1: the converging tube set up

At some position in the conical convergence (see figure 2.1) at cross-sectional diameter d_i, the static pressure is p_i and velocity v_i. According to the law of Bernoulli:

$$p_0 = p_i + \frac{1}{2} r\, v_i^2 \qquad (2.1)$$

or:

$$v_i = \sqrt{\frac{2}{r}}\, \sqrt{p_0 - p_i} = C_1 \sqrt{p_0 - p_i} \qquad (2.2)$$

and with the equation of continuity:

$$v_i\, d_i^2 = C_2 , \qquad (2.3)$$

the result becomes:

$$\sqrt{p_0 - p_i}\,.\, d_i^2 = Constant. \qquad (2.4)$$

Relationship (2.4) will be now verified by means of measurement and calculation.

Momentum balance:

Let us choose **l** and **u** as control surfaces. Because of the strong contraction out of the settling chamber, the velocity v_1 and the static pressure p_1 can be assumed constant in the cross section at **l**. The flow at the exit is taken to be adapted to the surrounding so that the pressure in the exit plane, p_u, is equal to the atmospheric pressure. Since all pressures are considered with respect to the atmospheric pressure, this means that $p_u = 0$.

The momentum balance over the two control surfaces then leads to:

$$p\, r_1^2 (p_1 + r\, v_1^2) = F + \int_0^{r_u} r\, v_u^2 .2pr\, dr \qquad (2.5)$$

Here, F is the force, directed to the left, which is exerted by the conical section on the flow. This force is given by:

$$F = \int_{r_u}^{r_1} p_i\; 2p\; r_i . dr_i \qquad (2.6)$$

For i=1, equation (2.1) results in:

$$p_0 = p_1 + \frac{1}{2} r\, v_1^2 \qquad (2.7)$$

With a total pressure tube, the stagnation pressure or total pressure in the exit cross section <u>*u*</u> is measured:

$$p_{tot} = p_u + \frac{1}{2} r v_u^2 = \frac{1}{2} r v_u^2 \qquad (2.8)$$

where we have used $p_u = 0$. With (2.6), (2.7) and (2.8), the momentum balance becomes:

$$r_1^2(2\,p_0 - p_1) = 2\int_{r_u}^{r_1} p_i r_i \; dr_i + 4\int_0^{r_u} (p_{tot}.r)dr \qquad (2.9)$$

(check the derivation).
The relationship (2.9) is checked in the experiment.

2.3 Experimental Set up

A fan blows air in the settling chamber. With help of variable transformer the number of revolutions can be adjusted. The diameter of the settling chamber is big with respect to the diameter of the converging tube. The measured pressure in the settling chamber, p_0, can therefore be set equal to the total pressure of the flow. This total pressure is connected to the front end of the manometer on the big reservoir and at the back end, marked with an **A**. The measured pressure difference here is zero. This connection monitors the decrease in the liquid level of the big reservoir: due to the measurements, liquid is drawn from this reservoir. By means of this connection, it is possible to correct the measured pressure differences ($p_0 - p_i$) for this zero-shift.

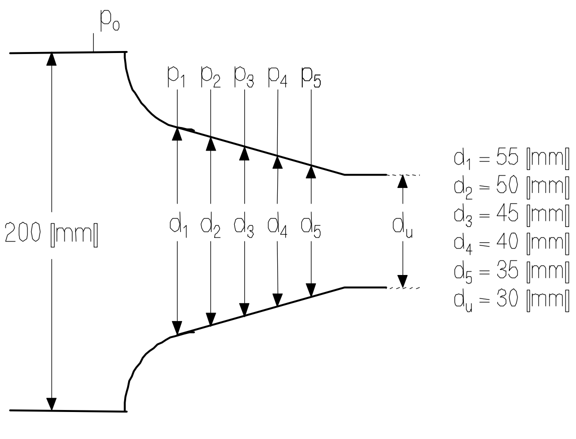


Figure 2.2: dimensions of the set up

In the converging tube, 5 measuring points for static pressure are installed at the diameters shown in figure 2.2. The static pressures, p_i, are connected to the back side of the manometer. A traversing total pressure tube, which can be traversed from one side of the exit plane to the other, can measure the flow at the exit. As mentioned before, the pressure that is registered, is the dynamic pressure: ½ $r\ v_u^2$.

2.4 Carrying out the Experiment

During the experiment, do not block the exit of the set up. The pressure in the settling chamber may become too high if the air jet is blocked and water will come out of the manometer.

1) Set p_0 to approximately 20-mm water column, i.e. 10 [cm] on the scale. This value should be kept constant during the experiments.

2) Measure the five pressure differences $p_0 - p_i$. Determine the zero-shift by switching the fan on and off. The shift is quite small and it will take some time to establish.

3) Determine the middle position of the total pressure tube. This can be done by measuring two symmetrical positions: the tube is traversed into the boundary layer, so that the pressure will be 10 mm water column (5 cm on the scale). The velocity gradient is high in this region, so the position of the tube can be determined accurately (~0.01 mm). Repeat this procedure at the other side of the exit and determine the mean position, which should be the exact middle position. Place the tube in the middle and traverse to the wall, so the pressure profile is determined. Up to approx. 13 mm from the middle, the total pressure is nearly constant. In the boundary layer (13 mm to 15 mm from the middle), the step size should be 0,25 mm.

2.5 Questions

a) What is the theoretical velocity at the wall?
b) Is this in agreement with the measurement? If not, explain the difference.
c) What value should be used in the calculations, the theoretical or the experimental? Why?

Check relationship (2.4) (which was derived from the Bernoulli law) by calculating the constant six times (i.e. for i = 1,2,3,4,5 and u). Do this with and without zero-shift correction. Should this correction be added or subtracted from the measured values? What is the maximum deviation (in %) of the mean, with and without correction?

Calculate the terms of the momentum balance (2.9). The first integral in the right hand side can be calculated as follows:

From (2.4):

$$\sqrt{p_0 - p_i}\,.r_i^2 \;=\; constant \;=\; \sqrt{p_0}\,.r_u^2\,, \qquad (2.10)$$

and therefore:

$$p_i \;=\; p_0\,[1 \;-\; (r_u\,/\,r_i)^4\,]. \qquad (2.11)$$

This can be substituted in the integral:

$$2\int_{r_u}^{r_1} p_i r_i \; dr_i = 2\,p_0\left(\int_{r_u}^{r_1} r_i \; dr_i - r_u^{\,4}\int_{r_u}^{r_1}\frac{d\,r_i}{r_i^3}\right) =$$

$$= 2\,p_0\left[\frac{(r_1^2 - r_u^2)}{2} - r_u^4\left(\frac{1}{2\,r_u^2} - \frac{1}{2\,r_1^2}\right)\right] =$$

$$= p_0\left(r_1^2 - 2\,r_u^2 + \frac{r_u^4}{r_1^2}\right) = p_0\left[\frac{(r_1 + r_u)(r_1 - r_u)}{r_1}\right]^2$$

so:

$$2\int_{r_u}^{r_1} p_i r_i . dr_i = p_0\left[\frac{(r_1 + r_u)(r_1 - r_u)}{r_1}\right]^2 \qquad (2.12)$$

The second integral in the right hand side will be determined numerically by means of the Simpson formula. First, calculate the part of the integral for which $p_{tot} \approx p_0$ (the straight part of the profile) and subsequently the remainder by means of the Simpson rule:

$$\int_0^{r_u}(p_{tot}.r)dr = p_0\int_0^{r_g} r\,dr + \int_{r_g}^{r_u}(p_{tot}.r)dr = \frac{1}{2}p_0 r_g^2 + I_{Simpson} \qquad (2.13)$$

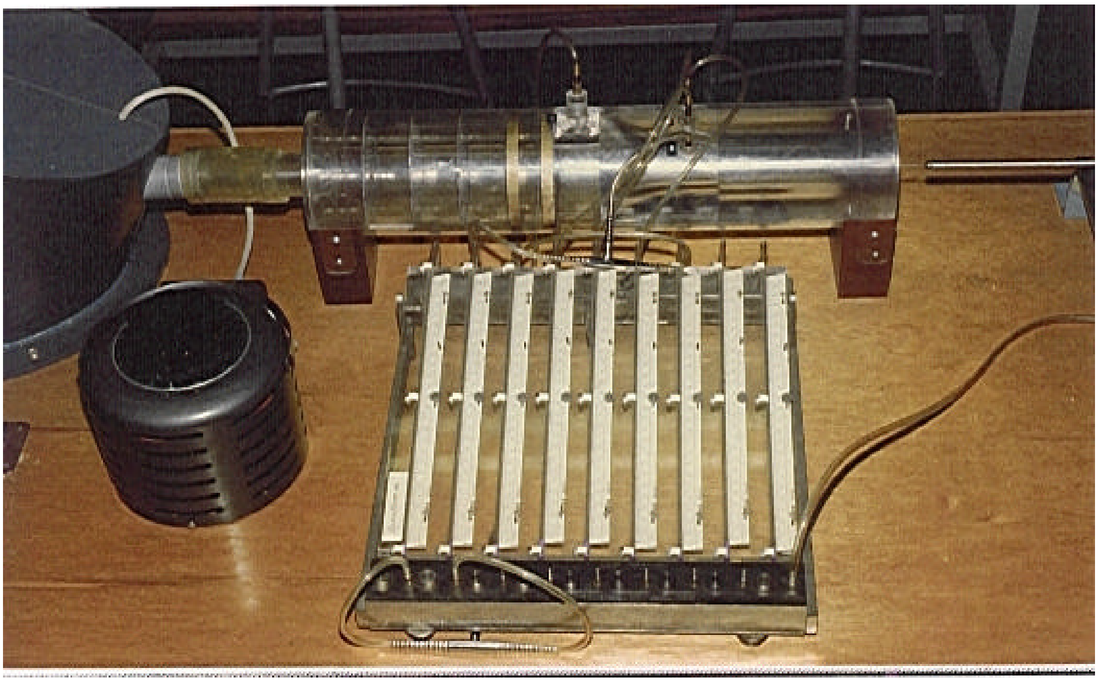
In this formula, r_g is the value of r at the beginning of the boundary layer. The Simpson formula, for an odd number of steps, is given by:

$$\int_{r_g}^{r_u} f(r)dr = \frac{h}{3}(f_1 + 4f_2 + 2f_3 + 4f_4 + 2f_5 + \ldots + 4f_{n-1} + f_n) \qquad (2.14)$$

Here, h is the step size, $f_1, f_2, \ldots, f_n$ are the measured values of ($p_{tot} \cdot r$) for the values of r ($r_g < r < r_u$). For an even number of steps, the Simpson rule is given by:

$$I_{Simpson} = \frac{h}{3}(f_1 + 4f_2 + 2f_3 + \ldots + 4f_{n-2} + \frac{5}{2}f_{n-1} + \frac{3}{2}f_n) \qquad (2.15)$$

If the measurements are done with reasonable care, the balance should add up with approximately 2 or 3 % error.



Experimental Set Up 3

Chapter 3
Flow through an abrupt expansion

3.1 Introduction

The aim of this set up is the investigation of the flow through an abrupt expansion (i.e. jump in diameter). The loss of mechanical energy after an abrupt expansion is illustrated. This loss is due to the fact that mechanical energy in dissipated into thermal energy. Extra attention is paid to the pressure in the plane of the expansion, to see the effect of back flow at the wall, caused by a ring vortex.

3.2 Theory

Quantity	Symbol	Units
Density of air	ρ_a	[kg/m^3]
Density of water	ρ_w	[kg/m^3]
Velocity	v	[m/s]
Cross-section diameter	d	[m]
Radius	r	[m]
Wall shear stress	τ	[N/m^2]
Pressure	p	[N/m^2]
Total pressure	p_{tot}	[N/m^2]
Static pressure	p_{st}	[N/m^2]
Dynamic pressure	Dp (=p_{tot} - p_{st} =½rv^2)	[N/m^2]
Atmospheric pressure	p_{atm}	[N/m^2]
Difference in height	Dh	[mm]
Integration step size	h	[m]
Specific heat (const. p)	c_p	[J/kgK]

Table 3.1: quantities involved

Air flows from a narrow tube into a wider one. Closely behind the expansion, a weak vortex ring is formed which results, near the wall, in a reversal of the direction of the flow. Further downstream, the flow will return over the entire cross section in the direction of the exit (**2**).

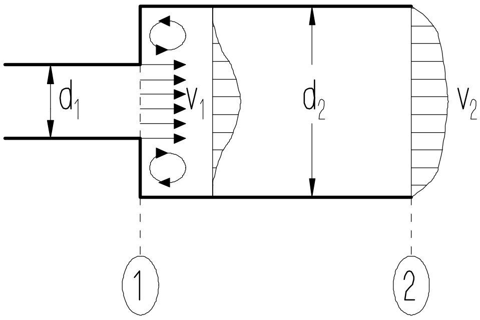


Figure 3.1: schematic drawing of the abrupt expansion

Mass balance
If the air is assumed to be incompressible, the mass balance over the cross sections **1** en **2** (see figure 3.1) is given by:

mass per sec through **1** = mass per sec through **2**

or

$$\int_0^{r_1} r\ v_1 .2pr\ dr\ =\ \int_0^{r_2} r\ v_2 .2pr\ dr \qquad (3.1)$$

and since v_1 = constant over the entire cross section:

$$\frac{1}{2} v_1 r_1^2\ =\ \int_0^{r_2} v_2 r\ dr \qquad (3.2)$$

Momentum balance
When wall shear stress is neglected and when it is assumed that the pressure at the outside tube surface at **1** is equal to the static pressure p_1 at the exit of the narrow tube, the momentum balance is given by:

force to the right = momentum flow through **2** – momentum flow through **1**

or

$$p\ r_2^2 (p_1 - p_2)\ =\ \int_0^{r_2} r\ v_2 . v_2\ 2pr\ dr\ -\ \int_0^{r_1} r\ v_1 . v_1\ 2pr\ dr \qquad (3.3)$$

or

$$(p_1 - p_2) r_2^2\ =\ r\ [2 \int_0^{r_2} v_2^2 r\ dr\ -\ v_1^2 r_1^2] \qquad (3.4)$$

Energy balance

The energy balance is given by:

kinetic energy through **1** + work pressure forces = kinetic energy through **2** + dissipation

or

$$\int_0^{r_1} \frac{1}{2}\, r\, v_1^2 . v_1\ 2pr\, dr + p_1 \int_0^{r_1} v_1\ 2pr\, dr - p_2 \int_0^{r_2} v_2\ 2pr\, dr =$$

$$= \int_0^{r_2} \frac{1}{2}\, r\, v_2^2 . v_2\ 2pr\, dr + y \qquad (3.5)$$

In this equation, Y is the dissipation. With the mass balance and the fact that v_1 = *constant*, this can be written as:

$$\frac{1}{2}\, r\, v_1^3 r_1^2 + (p_1 - p_2) v_1 r_1^2 = r \int_0^{r_2} v_2^3 r\, dr + \frac{y}{p} \qquad (3.6)$$

The aim of this experiment is to determine the terms in this energy balance, i.e. three terms by measurement and the dissipation Y from the balance between all terms. From Y, the temperature rise can be calculated.

3.3 Experimental Set Up

A fan blows air into a settling chamber. The number of revolutions can be set by means of a variable transformer. In the settling chamber, a number of gauzes and grids ensure a homogeneous velocity and pressure distribution. The static pressure in the settling chamber is measured at position **a**. Because of the fact that the velocity in the chamber is quite low, the static pressure can assumed to be equal to the total pressure of the flow. The static pressure in the narrow tube, p_1, can be measured at connection **b** (remember all pressures are computed with respect to the atmospheric pressure).

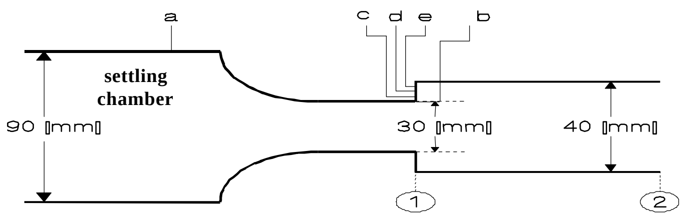


Figure 3.2: the set up of the abrupt expansion

The connections **c**, **d** and **e** give the pressure at different positions on the ring-shaped wall in **1**. These can be used to check the assumption that the pressure over the wall is constant in the plane of the expansion. At the exit of the wide tube, the static pressure, p_2, can assumed to be equal to the atmospheric pressure. Because the total pressure tube measures the total pressure with respect to the atmospheric pressure, it measures the dynamic pressure directly:

$$\Delta\ p_2 \ = \ (\ p_{tot} - p_{st}\ )_2 \ = \ p_{tot,2} \ = \ \frac{1}{2}\ r\ v_2^2 \qquad (3.7)$$

By traversing, the entire velocity profile in cross section **2** is measured. Because the static pressure in the settling chamber is equal to the total pressure in both the chamber and the cross section **1**, the velocity in this cross section, v_1, can be determined from:

$$\Delta\ p_1 \ = \ (\ p_{tot} - p_{st}\ )_1 \ = \ p_a - p_b \ = \ \frac{1}{2}\ r\ v_1^2 \qquad (3.8)$$

The velocity v_1 can also be determined from the difference between $p_{tot,1}$, measured by the total pressure tube (by sliding it into the tube) and the static pressure in **1**, measured by **b**.

The pressure is measured in mm water column; so if the dynamic pressure is equal to $\frac{1}{2} r_a v^2 = Dp = r_w gDh$, the velocity is given by:

$$v \ = \ \sqrt{\frac{2}{r_l}\ r_w\ g\Delta h.10^{-3}} \ \approx \ 4\sqrt{\Delta h} \qquad (3.9)$$

with ρ_a = 1.2 [kg/m^3], ρ_w = 1000 [kg/m^3] and g = 9.81 [m/s^2].

In other words:

$$v \ = \ 4\sqrt{\Delta p} \qquad (3.10)$$

with $\Delta p = (p_{tot} - p_{st})$ in [mm water column].

3.4 Carrying out the Experiment

During the experiment, do not block the exit of the set up. The pressure in the settling chamber may become too high if the air jet is blocked and water will come out of the manometer.

1) Set the variable transformer so that the pressure in the settling chamber is 9 mm water column above atmospheric pressure. This pressure may rise slightly during the experiment (e.g. after sliding the total pressure tube in). It should then be brought back to its original setting.

2) In the momentum balance, it was assumed that over the entire ring surface in **l**, the pressure was constant and equal to p_1. This means that $p_b = p_c = p_d = p_e$. Measure these pressures with respect to the atmospheric pressure (mind the sign!).

3) It can be assumed that the velocity profile at the exit of the narrow tube is flat; i.e. the entire cross section has the same velocity. Measure this velocity, both by determining the difference $p_a - p_b$ and by sliding the total pressure tube into the wide tube. If necessary,

adjust to the flow with the transformer. The tube measures $p_{tot,1}$, so p_1 (= p_b) has to be subtracted. Are the values of $p_b - p_e$ equal to those before inserting the tube and the correction? Which method to determine v_1 will be the most accurate?

4) Determine the velocity profile in cross section **2**. The determination of the centre position can not be done in the same manner as in experiment 2, because the profile is rather flat in this case. It is therefore advisable to start at the wall and measure the entire profile in steps of 5 mm; in this way, two values for r = 20, 15, 10 and 5 are obtained, which can be averaged. Make sure that the step size is kept constant, because of the numerical integration.

3.5 Questions

a) The influence of the wall shear stress was neglected. If this had not been done, what would be the influence on the mass, momentum and energy balances?

b) Sketch the wall shear stress as a function of the position in axial direction in the wide tube, between **1** and **2**.

c) When the abrupt expansion is replaced by a more gradual transition (e.g. a diffuser), the loss in mechanical energy can be reduced remarkably; explain this. What are the constraints for such a diffuser?

From the integral at the right hand side of (3.6), it can be seen that (v^3r) is a measure for the kinetic energy flow, just as v is a measure for the volume flow. Plot the velocity and energy (v^3r) profiles for cross sections **1** and **2**.

Calculate the terms in the energy balance (3.6). The difference in the static pressures in the cross sections **1** and **2**, ($p_1 - p_2$), has to be converted from [mm water column] to [N/m^2]. N.B.: 1 mm WK ≅ 9.81 [Pa].

The integration of

$$\int_0^{r_2} v_2^3 r \, dr \tag{3.11}$$

is done numerically, by means of the Simpson formula:

$$\int_0^{r_2} f(r)\, dr = \frac{h}{3}(f_1 + 4 f_2 + 2 f_3 + 4 f_4 + 2 f_5 + \ldots + 4 f_{n-1} + f_n) \tag{3.12}$$

This formula is valid for an odd number of observations, n. h is the step size and $f_1 - f_n$ are the measured values of v_2^3r for $0 > r > r_2$.). For an even number of steps, the Simpson rule is given by:

$$I_{Simpson} = \frac{h}{3}(f_1 + 4 f_2 + 2 f_3 + \ldots + 4 f_{n-2} + \frac{5}{2} f_{n-1} + \frac{3}{2} f_n) \qquad (3.13)$$

Assume that the dissipated part of the mechanical energy is completely transferred into heat. The temperature rise of the air can be calculated from:

$$y = \Phi_m c_p \Delta T \qquad (3.14)$$

In this equation, c_p is the specific heat at constant pressure of air ($c_p = 10^3$ [J/kgK]). The mass flow Φ_m can easily be calculated in cross section 1, because here the velocity is constant so that:

$$\Phi_m = r\, v_1 . p\, r_1^2 \qquad (3.15)$$