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Implementation of Boundary Conditions on an Irregular Geometry by Extrapolation to the Staircase Boundary of a Cartesian Grid

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A convenient way to study wave-related processes in coastal and harbor applications is by means of numerical simulation. Examples of problems that are considered are the wave load on coastal defense systems, wave-induced currents and sediment transport in the surf zone, wave penetration in harbors, and harbor oscillations. Typical of these applications is that waves and wave dynamics relevant to the problem are present almost everywhere in the domain. Numerical modeling therefore requires a fairly uniform grid-point resolution across the entire domain, regardless of the wave model that is used. This makes it quite attractive to use Cartesian grids: the use of a Cartesian grid simplifies the numerical method and the software implementation, allows the use of efficient solution techniques that exploit the regular grid structure, and facilitates the use of higher-order central discretizations. Central discretizations are essential to avoid an unacceptable loss of wave energy over large distances. Higher-order discretizations provide a substantially higher accuracy given a certain grid resolution and hence reduce the number of grid points and amount of work required to obtain a certain accuracy. A solution needs to be found, however, for how to deal with the intersection of grid lines by arbitrarily oriented boundaries such as those representing harbor walls.

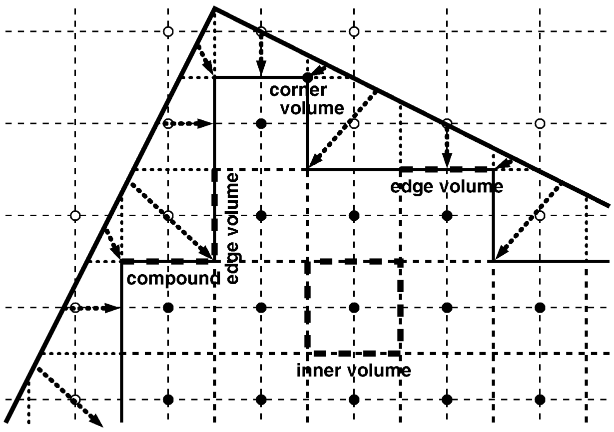
A suitable way to describe the dynamics of free-surface waves in the shallow waters of coastal zones and harbors (water depths up to about 20 to 30 meter) is by means of a Boussinesq-type wave model. We have developed such a model at WL | Delft Hydraulics (now part of the new organization Deltares), using a Cartesian grid, a cell-vertex finite-volume technique, central space discretizations, and a four-stage Runge-Kutta time integration method (Borsboom *et al.*, 2000). The conditions to be imposed at boundaries can be fairly complex, because of the subgrid effects that they must include. For example, a Boussinesq-type model is not suitable to model in detail the wave breaking and wave run-up on a breakwater. On the other hand, the details of these aspects of wave dynamics, which are at a scale of several (tens of) meters, are often irrelevant in wave studies involving typically an area of the order of a kilometer squared. The *effect* of these aspects on the overall wave dynamics, in particular the effect of the partial reflection of waves, can be important in certain applications, however, and are then to be included in the boundary condition applied at structures (Borsboom *et al.*, 2001).

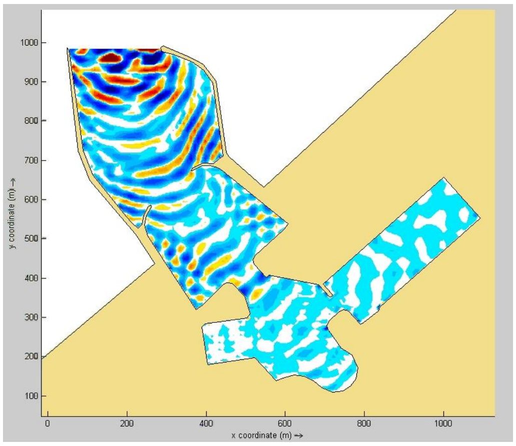
A technique is required to implement the complex conditions at immersed boundaries. One possibility is to use a cut-cell technique (see, e.g., Ingram et al., 2003), which can be viewed as a method where ‘the variables are brought to the boundaries’ by means of extrapolation/interpolation. We found that this method has several disadvantages: interpolations may have to be adapted to ensure stability, accurate discretization of the model equations on cut cells is not trivial, and special measures may be required to deal with very small cut cells.

The last two disadvantages are not present if the boundary of the computational domain would be the staircase boundary enclosing the inner finite volumes, which are the 2D finite volumes that lie inside the oblique physical boundary (cf. the left figure). Each inner volume surrounds an inner grid point, indicated by the solid circles. Along the staircase boundary we discern 1D edge volumes along straight parts, 1D compound edge volumes at inner corners, and 0D corner volumes at outer corners. Each boundary volume is associated with a virtual grid point, indicated by the open circles. Because all boundary volumes are in between grid points, accurate central discretizations are possible for both 0th and 1st derivative terms.

Obviously, the position and orientation of the staircase boundary should *not* be used for imposing the boundary conditions. However, it is quite straightforward to expand the conditions imposed at the true boundary to the staircase boundary where the expanded conditions can be discretized (cf. the arrows in the left figure). Because the staircase boundary is *inside* the true boundary, this automatically leads to stable extrapolations. This approach, where 'the boundaries are brought to the variables', has the simplicity of a staircase boundary while preserving the position and orientation of the true boundary. A drawback is that, since the model equations are not applied in the gap between the staircase boundary and the true boundary, strict conservation of, e.g., mass is lost. We think that it is possible to correct for this, but have never found the need to develop that further.

The presented alternative cut-cell method has been implemented in the Boussinesq-type wave model mentioned above. Details of the implementation will be presented at the colloquium. A typical application and a typical result are shown in the figure on the right.





Left figure: principle of applied boundary condition implementation.
Right figure: wave simulation in Scheveningen harbor.

References

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