An Immersed Boundary Method for Computing Anisotropic Permeability of Structured Porous Media

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Introduction

Many porous media exhibit a strong anisotropy in their mechanical and thermal properties. An example of such a porous medium is formed by a system of aligned cooling rods in large-scale heat-transfer devices. On much smaller scales anisotropic porous media may function as carriers for catalyst material in micro-reactors. In this contribution we set out to compute the macroscopic transport coefficients of anisotropic porous media on the basis of fully resolving the flow that passes through them. This provides a clear point of reference for accurate coarsened (macroscopic) flow descriptions that can be used in engineering applications.

By simulating the laminar flow through porous media in all detail, assuming a given topology of the porous region, we can obtain the so-called ‘permeability tensor’ from first principles. Specifically, we will study porous media composed of a family of rectangularly shaped, three-dimensional rods (cf. Fig. 1), and calculate their permeability as a function of the fluid velocity and the porosity. We show results obtained using an ‘immersed boundary’ method with which the intricate geometry of the porous medium is represented.

Macroscopic Transport Modeling

Traditionally, computational engineering predictions of transport processes in porous media applications are based on a ‘macroscopic’ formulation of the governing equations. The fundamental starting point for these macroscopic equations are the transport equations that resolve all processes down to the ‘pore scale.’ By spatially coarsening these fundamental equations over a control volume that is large compared to the pore size, but much smaller than the macroscopic length scale, the volume-averaged transport equations can be derived (Saito & de Lemos [3]). The volume-averaging technique typically introduces new terms such as the (scalar) porosity, which is the ratio of the volume which the fluid occupies to the total control volume, and the permeability tensor $K$. The permeability is a measure of the resistance felt by the fluid flowing through the porous material.

Permeability is a key ingredient in engineering models for predicting the macroscopic flow behavior. The permeability is dependent on the internal geometry of the porous medium and on the operating flow conditions. In general, some degree of phenomenological modeling is required in order to close the governing macroscopic equations on the level of $K$. Herein lies the potential difficulty with predicting macroscopic flow – assumptions need to be made on several of the mechanisms that influence the tensor $K$ that are not all well controlled. By simulating the flow through examples of porous media in full detail, it is possible to quantify the tensor-components of $K$, and in this way, use a first principle’s approach to underpin permeability modeling.

Computational Setting

The permeability tensor will be obtained numerically by modeling a porous medium by a set of rectangular rods placed in regular patterns. The full set of equations will be solved in an elementary volume extended with periodic boundary conditions (Kuwahara, et al. [1]). These representative elementary volumes (REVs) provide an accurate local assessment of the flow’s resistance anisotropy in porous media. We will consider the resulting pressure drop that arises from imposing a specific mass-flow rate in different directions through the REV. By determining the dependence of pressure drop on flow rate and flow direction in separate simulations, it will be possible to extract the anisotropic permeability without further approximation.

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Numerical Modeling

The numerical simulations will be carried out on the basis of the incompressible Navier-Stokes equations. We impose periodic boundary conditions on all flow variables, except the pressure. The pressure will be decomposed into a periodic field and a linearly varying mean pressure field, which is used to enforce a given mass-flow rate. As for the solid boundaries, their presence will be simulated with the help of the immersed boundary (IB) method (Mittal & Iaccarino [2]). Therefore, the equations governing fluid dynamic behavior in dimensionless form are:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f},
\]

with \(\mathbf{f}\) the momentum forcing term. The forcing term contains information about the presence of 'fluid-submerged' bodies. That is, by providing a local momentum forcing, simulating the net momentum exchange between fluid and solid, various boundary conditions can be applied on their mutual interfaces.

Various forms of the IB method exist. In this contribution we first consider a simple 'masking function' technique in which

\[
\mathbf{f} = -\frac{1}{\epsilon} H(x) (\mathbf{u} - \mathbf{u}_s),
\]

and the scalar masking function \(H\) equals 1 inside regions of the domain that are occupied by the solid material and 0 elsewhere. The positive parameter \(\epsilon \ll 1\) controls the effectiveness of the forcing, i.e., decreasing \(\epsilon\) causes an increase in the magnitude of \(\mathbf{f}\) and therefore a rapid decrease in the velocity difference \(\mathbf{u} - \mathbf{u}_s\). As the magnitude of the forcing term can have an adverse effect on the stability of the numerical scheme its contribution to the total flux is treated implicitly in the time-stepping approach. In the end, this IB formulation allows for a Dirichlet-type boundary condition to be approximated in which the solid boundary moves with a velocity \(\mathbf{u}_s\).

For the numerical discretization, we adopt a symmetry preserving finite volume formulation and treat all other contributions to the flux using an explicit time-stepping method (Verstappen & Veldman [4]).

Discussion

In Fig. 2(a) we present a REV of an example porous medium with cross-sectional planes displaying the streamwise velocity component. Clearly visible from Fig. 2(b) is the ability of the IB method to capture near-wall dynamics. In the full paper we will focus on permeability tensor-component predictions of various porous topologies at varying Reynolds numbers and porosities. Our aim is to produce accurate local anisotropic permeability correlations for use in macroscopic formulations of the governing momentum equations. We will also perform boundary layer analysis, comparing the IB method with standard treatments for the no-slip condition.

References