Ghost-cell immersed boundary method for inviscid compressible flows

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Overview

- Introduction and background
- Potential flows
- Euler flows
- Conclusions

Outline

Motivation

'Vision 2020': studying and minimizing the aviation industry's impact on the global climate. Reductions in:

- Fuel consumption
- Noise
- Emissions

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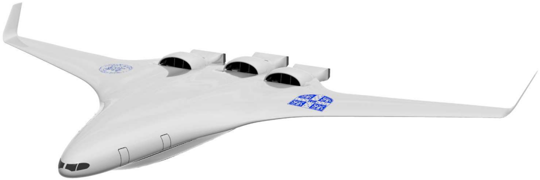
Development of unconventional aircraft configurations



Goal

A computational method for preliminary aerodynamic design and optimization of unconventional aircraft configurations.

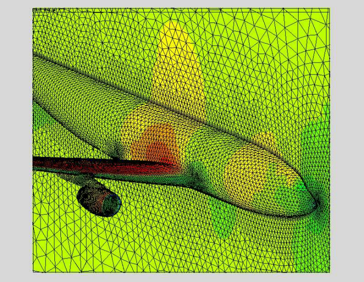
Requirements:

- Fast (solving and mesh generation)
- Accuracy similar/better than codes currently used in optimization
- Handle ‘complex’ geometries

Immersed boundary method

Ghost-cell method for full-potential equation and Euler equations:

- No mesh generation
- Straightforward implementation
- Fast solution
- Able to describe shock waves



Outline

Full potential equation

- Governing equation (conservative):

$$(\rho\phi_x)_x + (\rho\phi_y)_y = 0,$$

or (non-conservative):

$$(a^2 - u^2)\phi_{xx} - 2uv\phi_{xy} + (a^2 - v^2)\phi_{yy} = 0.$$

- Simple test equation for ghost-cell method on compressible flows.
- Error in shock jumps small until $M \approx 1.3$.
- Fast solution with Approximate Factorization algorithm.

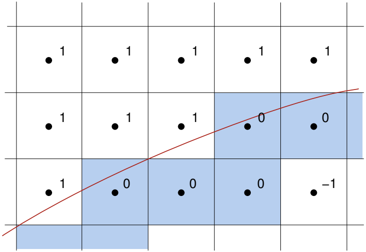
Ghost-cell method

- Identification ghost cells and fluid cells.
- Boundary condition:

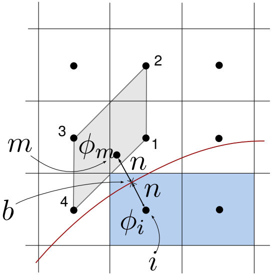
$$\vec{u} \cdot \vec{n} = 0 \quad \rightarrow \quad \frac{\partial \phi}{\partial n} = 0.$$

- Set value in ghost cell:

$$\phi_i = \phi_m.$$



(c)



(d)

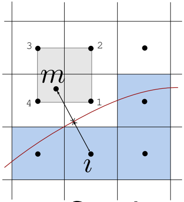
Bilinear interpolation - case I

Assume $\phi(x, y) = a + bx + cy + dxy$. With four surrounding points:

$$\begin{pmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \\ 1 & x_4 & y_4 & x_4 y_4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}.$$

value of ϕ_m follows from:

$$\phi_m = a + bx_m + cy_m + dx_m y_m.$$

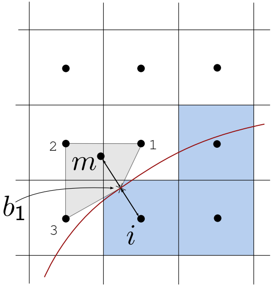


Case I

Bilinear interpolation - case II

Use $\frac{\partial \phi}{\partial n} = 0$ in point b_1 by differentiating the bilinear form:

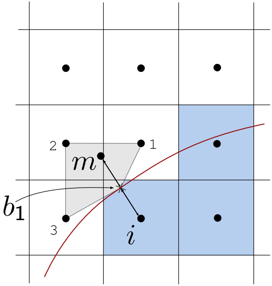
$$\frac{\partial \phi}{\partial n} = b\frac{\partial x}{\partial n} + c\frac{\partial y}{\partial n} + d\left(x_b\frac{\partial y}{\partial n} + y_b\frac{\partial x}{\partial n}\right) = 0,$$



Case II

Bilinear interpolation - case II

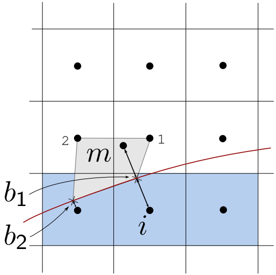
$$\begin{pmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \\ 0 & \left(\frac{\partial x}{\partial n}\right)_{b_1} & \left(\frac{\partial y}{\partial n}\right)_{b_1} & x_{b_1}\left(\frac{\partial y}{\partial n}\right)_{b_1} + y_{b_1}\left(\frac{\partial x}{\partial n}\right)_{b_1} \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ 0 \end{pmatrix}.$$



Case II

Bilinear interpolation - case III

$$\begin{pmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 0 & \left(\frac{\partial x}{\partial n}\right)_{b_1} & \left(\frac{\partial y}{\partial n}\right)_{b_1} & x_{b_1}\left(\frac{\partial y}{\partial n}\right)_{b_1} + y_{b_1}\left(\frac{\partial x}{\partial n}\right)_{b_1} \\ 0 & \left(\frac{\partial x}{\partial n}\right)_{b_2} & \left(\frac{\partial y}{\partial n}\right)_{b_2} & x_{b_2}\left(\frac{\partial y}{\partial n}\right)_{b_2} + y_{b_2}\left(\frac{\partial x}{\partial n}\right)_{b_2} \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ 0 \\ 0 \end{pmatrix}.$$



Case III

Results

- NACA 0012; 400 points to describe surface.
- $M_\infty = 0.8$, $\alpha = 0^\circ$.
- Domain 11×5, stretched grid.

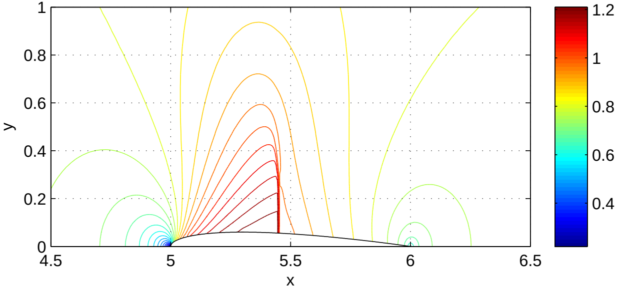
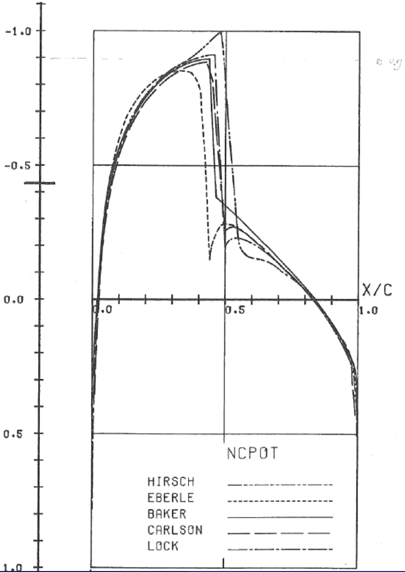


Figure: Mach contour lines, $\Delta x = \Delta y = 1/200$.

Results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HIRSCH
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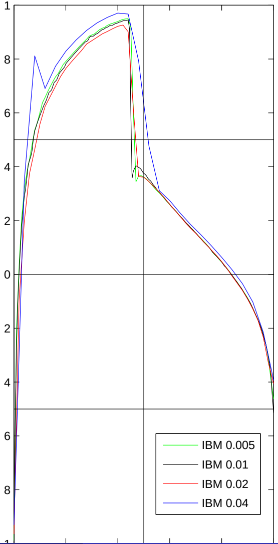
BAKER
CARLSON
LOCK


IBM 0.005
IBM 0.01
IBM 0.02
IBM 0.04

Outline

Governing equations

Unsteady, 2D inviscid flow:

$$\frac{\partial \boldsymbol{q}}{\partial t} + \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial x} + \frac{\partial \boldsymbol{g}(\boldsymbol{q})}{\partial y} = 0 \tag{1}$$

- Finite-volume discretization:

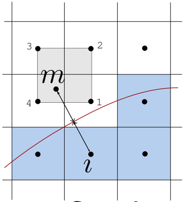
$$\Omega_{ij} \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = \left(\boldsymbol{F}_{i-1/2,j} - \boldsymbol{F}_{i+1/2,j}\right) \Delta y_j + \left(\boldsymbol{G}_{i,j-1/2} - \boldsymbol{G}_{i,j+1/2}\right) \Delta x_i.$$

- MUSCL with minmod limiter to reconstruct $\boldsymbol{q}^l_{i+1/2,j}$ and $\boldsymbol{q}^r_{i+1/2,j}$.
- Fluxes with HLL or exact Riemann solver.
- RK3 time discretization (TVD).

Ghost-cell method

Four boundary conditions are needed (Dadone & Grossman, 2004):

1. $u_n = 0 \quad \longrightarrow \quad u_{n,i} = -u_{n,m},$

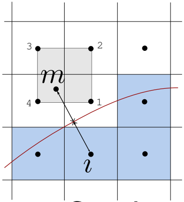


Case I

Ghost-cell method

Four boundary conditions are needed (Dadone & Grossman, 2004):

1. $u_n = 0 \quad \longrightarrow \quad u_{n,i} = -u_{n,m},$
2. $\dfrac{\partial p}{\partial n} = \left(\dfrac{\rho u_t^2}{R}\right)_b \quad \longrightarrow \quad p_i = p_m + 2\Delta n \left(\dfrac{\rho u_t^2}{R}\right)_b,$



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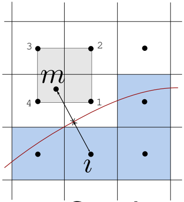
3. $\dfrac{\partial s}{\partial n} = 0 \quad \longrightarrow \quad \rho_i = \rho_m \left(\dfrac{p_i}{p_m}\right)^{1/\gamma},$



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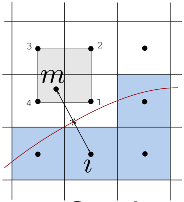
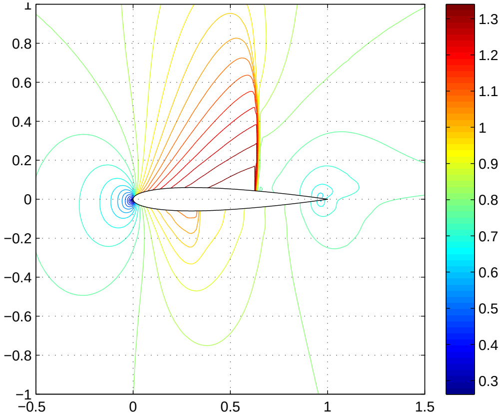
4. $\dfrac{\partial H}{\partial n} = 0 \quad \longrightarrow \quad u_{t,i}^2 = u_{t,m}^2 + \dfrac{2\gamma}{\gamma - 1}\left(\dfrac{p_m}{\rho_m} - \dfrac{p_i}{\rho_i}\right).$



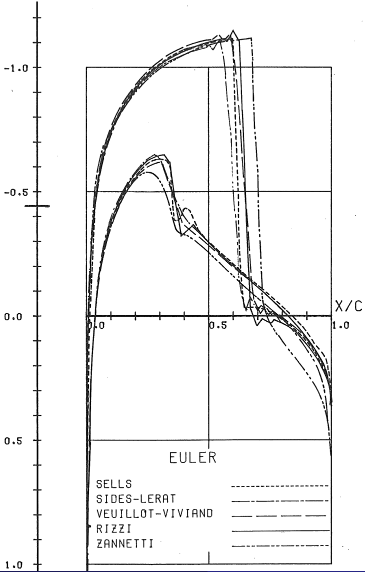
Case I

Results

- NACA 0012; 800 points to describe surface.
- $M_\infty = 0.8$, $\alpha = 1.25°$.
- Domain 21×20, stretched grid.



Results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0.0
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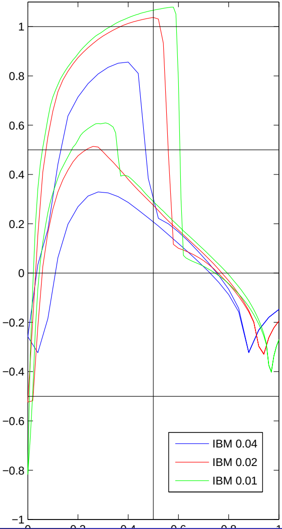
EULER
SELLS
SIDES-LERAT
VEUILLOT-VIVIAND
RIZZI
ZANNETTI


1
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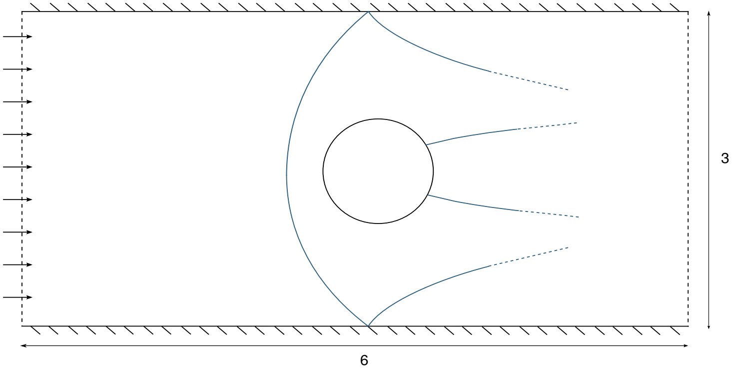
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
IBM 0.04
IBM 0.02
IBM 0.01

Unsteady supersonic flow over a cylinder at M=2



Outline

Conclusions

Ghost-cell immersed boundary method for inviscid flows:

- Useful method for design and optimization.
- General method for compressible flows in complex geometries.
- Accuracy similar to body-fitted grids.

Improvements and suggestions

- For steady flows: speed-up with multigrid.
- Thin body (e.g. trailing edge) treatment.
- Local grid refinement.
- Comparison with the same discretization on a body-fitted grid.
- Research into numerical entropy generation and conservation properties.
- Assess sensitivity needed for optimization.