Simulations of Turbulent and Transitional Flows using the Immersed Boundary Method

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“Immersed Boundary Methods: Current Status and Future Research”
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Immersed Boundary (IB) methods

Ever increasingly in popularity:
- 121,000 hits on Scholar.google.com
- 1,100 articles listed on Scopus since 1996
- 112 articles in Journal of Computational Physics
- 24 articles in Physics of Fluids
- 14 in Journal of Fluid Mechanics
- H-index = 45

One father (perhaps a few grandparents...): Charles Peskin, PhD Thesis, 1972

Reviews:
- Mittal & Iaccarino, Annual Review of Fluid Mechanics, 2005
  - General focus on methods
  - #6 most downloaded article (5732 times)
  - #19 most cited article (139 times)
  - General focus on simulation of turbulence
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- A variety of research areas:
  - Develop numerical methods
  - Study fluid mechanics in complex domain (perhaps with moving boundaries)
  - Interested in interactions between fluid and structures
  - Analyze multiphase flows
  - ...

How did I join the group?

I experienced IB's power during a summer program at Stanford in 1998 with Roberto Verzicco and Jamal Mohd-Yusof.

I was tired of meshing!

I wrote [with Frank Ham] a computer code called TOMmie.
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Generates Cartesian/Cylindrical grids with local, anisotropic refinement starting from raw CAD surfaces

- Airfoil leading edge
- Jet engine combustor
- Bluff body supersonic flows
Immersed Boundary Methods

Complex Geometries...

Flow around a coral reef

Flow is a jet engine combustion chamber
The simplest classification is:

- **Forcing methods**: define a suitable source term in the governing equation (continuous or discrete) to reproduce the effect of the *immersed* boundary

- **Reconstruction methods**: introduce the boundary condition directly in the discrete operators/stencils
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Are they really different?
The simplest classification is:

- **Forcing methods**: define a suitable source term in the governing equation (continuous or discrete) to reproduce the effect of the immersed boundary
- **Reconstruction methods**: introduce the boundary condition directly in the discrete operators/stencils

Are they really different?

Some (disputable) reasons to prefer reconstructions:

- Still solving the *original* governing equations
- The IB treatment *looks like* a special boundary condition
- You can do *funky* things at the boundaries, such as implement *wall models* for turbulent boundary layers
Two parts

1. Review the efforts at Stanford in using IB for *high-fidelity* simulations of turbulent flows

2. Illustrate recent attempts to use IB for hypersonic transitional boundary layer analysis
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The *missing* parts

- Complex geometries
- Reynolds-averaged modeling
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Acknowledgements:

Seongwon Kang recently graduated from Stanford
Olaf Marxen research associate
Simulations of Wall-Bounded Turbulent Flows

Flow around a thin airfoil, $Re_c = 1 \times 10^5$

Figure: Iso-surface of instantaneous vorticity
Simulations of Wall-Bounded Turbulent Flows

Why is it a challenge?

The reasons of the physics
The mechanism of vorticity generation at the wall has a fundamental importance in the flow dynamics.
The wall pressure fluctuations are a dominant source of acoustic disturbances.

The reasons of the numerics
The resolution requirement increases with the Reynolds number ($N \approx Re^{3/2}$).
Numerical dissipation at the wall completely alters the dynamics of the small scales.
Simulations of Wall-Bounded Turbulent Flows

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Simulations of Wall-Bounded Turbulent Flows

Computational Approach

Solve the Navier-Stokes equations for an incompressible fluid

\[
\frac{\partial u_i}{\partial x_i} = 0
\]

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{x_i^2}
\]

- Finite Volume 2nd in space and time
- unstructured grids with local refinement [Iaccarino & Ham, 2002]
Definitions of control volumes (CVs) near immersed boundaries

In the **approximate domain method** we formally define a boundary inside the actual flow domain $\Omega_{\text{fluid}}$.
The boundary condition is enforced via a reconstruction operator applied in the vicinity of the IB surface.

\[ u_{i,c} = \sum_{nb} w_{i,nb} u_{i,nb} + w_{IB} u_{i,IB} \]

The neighbors/weights included determine the formal accuracy of the approach:

- Linear 1D interpolation [Fadlun et al, JCP 2000]
- Linear MultiD interpolation [Majumdar et al, CTR Brief 2001]
- ... 
- Quadratic MultiD interpolation [Kang et al, AIAA 2009]

**Figure:** Reconstruction stencil for a staggered grid environment
We use an RK integrator (within a projection procedure) to solve the incompressible NS equations

\[
\frac{1}{\Delta t} - \frac{(\alpha_k + \beta_k)}{2Re} \frac{\partial^2}{\partial x_j \partial x_j} u_i^k = \frac{u_i^{k-1}}{\Delta t} - (\alpha_k + \beta_k) \frac{1}{\rho} \frac{\partial p^{k-1}}{\partial x_i}
\]

\[-\alpha_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-1} - \beta_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-2} + \frac{(\alpha_k + \beta_k)}{2Re} \left( \frac{\partial^2 u_i^{k-1}}{\partial x_j \partial x_j} \right)\]
We use an RK integrator (within a projection procedure) to solve the incompressible NS equations

\[
\begin{bmatrix}
\frac{1}{\Delta t} - \frac{(\alpha_k + \beta_k)}{2Re} \frac{\partial^2}{\partial x_j \partial x_j} \\
\end{bmatrix}
\]

\[
u_k^i = \frac{u_i^{k-1}}{\Delta t} - (\alpha_k + \beta_k) \frac{1}{\rho} \frac{\partial p^{k-1}}{\partial x_i} - \alpha_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-1} - \beta_k \left( \frac{\partial u_i u_j}{\partial x_j} \right)^{k-2} + \frac{(\alpha_k + \beta_k)}{2Re} \left( \frac{\partial^2 u_i^{k-1}}{\partial x_j \partial x_j} \right)
\]

It can be written \textit{discretely} as sums over neighbors of a cell

\[
\sum_{nb} \omega_{i,nb} u_{i,nb}^k = \sum_{nb} \omega_{i,nb} u_{i,nb}^{k-1} - \frac{\Delta t}{\rho} \frac{\partial p^{k-1}}{\partial x_i} + C_i^{k-2}
\]
Recall \( u_{i,c} = \sum_{nb} w_{i,nb} u_{i,nb} + w_{IB} u_{i,IB} \)

Is the reconstruction a boundary condition or *effectively* a wall model approximating the momentum equation

\[
\begin{align*}
  u_{i,c}^k &= \sum_{nb} w_{i,nb} u_{i,nb}^k + w_{IB} u_{i,IB}^k + \sum_{nb} w_{i,nb} u_{i,nb}^{k-1} + w_{IB} u_{i,IB}^{k-1} \\
  - \frac{\Delta t}{\rho} \left( \frac{\partial p^{k-1}}{\partial x_i} \bigg|_c - \sum_{nb} w_{i,nb} \frac{\partial p^{k-1}}{\partial x_i} \bigg|_{nb} - w_{i,IB} \frac{\partial p^{k-1}}{\partial x_i} \bigg|_{IB} \right).
\end{align*}
\]
Simulations of Wall-Bounded Turbulent Flows
Ensuring Mass Conservation

- Need to enforce global conservation of mass in the fluid domain:

\[ Q_{IB} = \int_{\Gamma_{IB}} \rho \vec{u} \cdot d\vec{A} = \sum_{m}^{N_{IB}} \rho u_{m,i} A_{m,i} \neq 0 \]
Simulations of Wall-Bounded Turbulent Flows
Ensuring Mass Conservation

- Need to enforce global conservation of mass in the fluid domain:

\[ Q_{IB} = \int_{\Gamma_{IB}} \rho \bar{u} \cdot dA = \sum_{m} \rho u_{m,i} A_{m,i} \neq 0 \]

- The reconstructed velocity \( u_{c,i} \) at the IB boundary does not necessarily satisfy conservation

- We write the formally \( \hat{u}_{c,i} = u_{c,i} + \delta u_{i} \) where the correction is defined to minimize the error (in mass conservation)

\[ J_{\lambda} = \sum_{m} \frac{1}{\omega_{m}} \left| \delta \bar{u}_{m} \right|^2 + \lambda \sum_{m} \rho \hat{u}_{m,i} A_{m,i} \]

\[ \omega_{m} \propto \left| x_{c} - x_{IB} \right| \]
Simulations of Wall-Bounded Turbulent Flows

Verification: Decay of Taylor vortex

Figure: Grid and IB configuration (a) and maximum error in $u_1$, $p$ and $T$ at $t=0.2$ (b) for a decaying vortex problem: ———, $u_1$; - - - - - - - - , $p$; · · · · · , $T$. 
Simulations of Wall-Bounded Turbulent Flows

The **unavoidable** $Re_\tau = 180$ channel flow

![Locally refined meshes for a turbulent channel flow; the size is $\approx 4$ million elements.](image)
Simulations of Wall-Bounded Turbulent Flows

The unavoidable $Re_T = 180$ channel flow

It’s very hard to generate an IB grid for a channel flow

- Periodicity
- Symmetry
- Wall resolution
Simulations of Wall-Bounded Turbulent Flows

The unavoidable $Re_\tau = 180$ channel flow

It’s very hard to generate an IB grid for a channel flow

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Figure: Locally refined meshes for a turbulent channel flow; the size is $\approx 4$ million elements.
Simulations of Wall-Bounded Turbulent Flows

The unavoidable $Re_\tau = 180$ channel flow

Figure: Mean streamwise velocity profiles in wall units: •, Kim et al. 1987; ———, inclined (IB) case; -----, body-fitted case.
Simulations of Wall-Bounded Turbulent Flows

The unavoidable $Re_\tau = 180$ channel flow

Figure: RMS velocity profiles in wall units: •, Kim et al., 1987; ---, inclined (IB) case; -- -- --, body-fitted case.
Simulations of Wall-Bounded Turbulent Flows

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Figure: Wall-pressure power spectra $\phi(\omega_t)$: ●, Choi and Moin, 1990; ———, inclined (IB) case; ———, body-fitted case; ·····, Cartesian collocated 10M mesh.
Simulations of Wall-Bounded Turbulent Flows

The unavoidable $Re_\tau = 180$ channel flow

**Figure:** Mean streamwise velocity profiles in wall units: •, inclined (IB) case; ---, stair-step approximation with $y^+ = yu_\tau / \nu$; ----, stair-step approximation with the down-shifted coordinate $y^+ = (y - k)u_\tau / \nu$. 

$$u/u_\tau$$

$$yu_\tau/\nu$$
Simulations of Wall-Bounded Turbulent Flows

Flow around a thin airfoil, $Re_c = 1 \times 10^5$

**Figure:** The locally refined mesh used in the present study ($\approx 7$ million elements) and a reference Cartesian mesh for the airfoil.
Simulations of Wall-Bounded Turbulent Flows

Flow around a thin airfoil, $Re_c = 1 \times 10^5$

**Figure**: Contours of the instantaneous streamwise velocity near the airfoil leading edge
Simulations of Wall-Bounded Turbulent Flows

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Figure: Averaged wall-pressure coefficients: ⊙, Roger and Moreau, 2004; ●, Wang et al., 2004; ×, IB method; △, body-fitted.
Figure: Wall-pressure power spectra at the trailing edge: ○, Roger and Moreau, 2004; ——, IB method; ---, body-fitted.
Simulations of Wall-Bounded Turbulent Flows

Conclusions of Part 1

“It’s hard but can be done!”

Careful attention has to be paid to grid resolution at the walls: Cartesian grids inherently sub-optimal for wall-bounded turbulent flows. Mass conservation and velocity reconstruction accuracy need to be at least compatible with the discretization accuracy.
Simulations of Wall-Bounded Turbulent Flows
Conclusions of Part 1

- “It’s hard but can be done!”
- Careful attention has to be paid to
  - Grid resolution at the walls: Cartesian grids inherently sub-optimal for wall-bounded turbulent flows
  - Mass conservation
  - Velocity reconstruction accuracy: needs to be at least compatible with the discretization accuracy
Simulations of Transitional Flows

Motivations

Thermal protection system (TPS) for planetary exploration

Berry & Hamilton, 2002

Apollo 11 at the Smithsonian
Simulations of Transitional Flows

Why is it a challenge?

All the reasons indicated before but also:

- The reasons of the physics
  - The critical quantities of interest (disturbances) have initially very small amplitudes
  - The transition process is very selective, only certain frequency range are unstable

- The reasons of the numerics
  - In addition to dissipation, we need to be careful about dispersion to preserve the nature of the disturbances
  - In high-speed boundary layers the amplification is very slow, thus the computational domains have to be very long!
All the reasons indicated before but also:
Simulations of Transitional Flows
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Simulations of Transitional Flows
Computational Approach

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\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho u_i u_j + p \delta_{ij} \right) = \frac{\partial \sigma_{ij}}{\partial x_j}, \quad i = 1, 2, 3,
\]

\[
\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j} \left[ (E + p) u_j \right] = -\frac{\partial q_j}{\partial x_j} + \frac{\partial}{\partial x_k} \left( u_j \sigma_{ij} \right)
\]

- High-order scheme preferable
- 6th order Compact Finite Difference [Lele 2005], 4th order time integration
Simulations of Transitional Flows
Discrete Roughness Elements

- We can generate locally refined meshes, as before

Body fitted grid

Immersed boundary grid
Simulations of Transitional Flows
Discrete Roughness Elements

- We can generate locally refined meshes, as before

- HOFD code can only use structured grids
- LOFV code can use locally refined grids
Simulations of Transitional Flows
Discrete Roughness Elements, $Ma = 4.9$

- The mean flow is easy...

Figure: Comparisons between body-fitted and immersed boundary computations for a 2D discrete roughness element
Simulations of Transitional Flows
Discrete Roughness Elements, $Ma = 4.9$

...the maximum amplification of small disturbances in a very challenging test!

Figure: Comparisons of density disturbance amplification between body-fitted and immersed boundary computations for a 2D discrete roughness element.
Simulations of Transitional Flows
Discrete Roughness Elements, $Ma = 4.9$

- ...the maximum amplification of small disturbances in a very challenging test!

The behaviour is qualitatively different downstream of the roughness
- BF $\rightarrow$ growth $\rightarrow$ unstable
- IB $\rightarrow$ decay $\rightarrow$ stable

**Figure:** Comparisons of density disturbance amplification between body-fitted and immersed boundary computations for a 2D discrete roughness element.
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Discrete Roughness Elements, $Ma = 4.9$

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- BF → growth → unstable
- IB → decay → stable

What is the role of the IB treatment?

**Figure:** Comparisons of density disturbance amplification between body-fitted and immersed boundary computations for a 2D discrete roughness element.
Volume forcing is incorporated using the feedback idea [Goldstein et al., 1993, Von Terzi et al., 2001] in the HOFD code:

$$F_i = \rho \int_{\Gamma_{IB}} f_i(x_{IB}) \cdot g(x - x_{IB}) \, dS$$

where

$$f_i = \alpha_S \cdot \int_0^t u_{i,IB}(x_{IB}, t') \, dt' + \beta_S \cdot u_{i,IB}$$

and $g$ is an approximation to the delta function:

$$g(x - x_{IB}) = \prod \exp \left( -\left( \frac{(x - x_{IB})}{\varrho} \right)^2 \right)$$
Simulations of Transitional Flows
Discrete Roughness Elements, $Ma = 4.9$

- Uniform grids in streamwise and spanwise direction

Body fitted grid

Immersed boundary forcing
Simulations of Transitional Flows
Discrete Roughness Elements, $Ma = 4.9$

- ...the comparison to BF results is much more convincing!

Figure: Comparisons between body-fitted and immersed boundary computations for a 2D discrete roughness element.
Simulations of Transitional Flows
3D Discrete Roughness Elements

- Ongoing work...

Figure: Flow structures and CVPs for a 3D discrete roughness element
Simulations of Transitional Flows
3D Distributed Roughness Elements

- Ongoing work...

Surface with distributed (interlaced wires) roughness

Simulations using the IB forcing method
“Prediction of transition to turbulence is perhaps the most challenging application of IB methods”
“Prediction of transition to turbulence is *perhaps* the most challenging application of IB methods”

Things did not work as planned...

- **High-order methods** are required to capture the transition region as the instabilities grow very slowly
- **Reconstruction** methods introduce numerical errors that alter the disturbance amplification
- **Forcing**: appears to provide accurate answers...but much more work is required to assess the validity of the present predictions
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