A Cartesian grid method using a direct discretization approach for simulations of heat transfer and fluid flow

Norikazu SATO 1, Takeo KAJISHIMA 2, Shintaro TAKEUCHI 2, Masahide INAGAKI 1 and Nariaki HORINOUCHI 1

1 Toyota Central R&D Labs., Inc., 41-1 Yokomichi, Nagakute, Aichi, Japan
nsato@mosk.tytlabs.co.jp
2 Osaka University, 2-1 Yamadaoka, Suita, Osaka, Japan

INTRODUCTION

In the automobile industry, computational fluid dynamics (CFD) has been extensively used at various engineering stages including fundamental studies and product design processes in the related areas to heat transfer and fluid flow problems. Although body conforming structured and unstructured grids are commonly used in those simulations, mesh generation especially for complex geometries is always a crucial issue for shortening the duration of a total analysis cycle. The Cartesian grid method, in which underlying grids do not need to coincide with body geometries, is expected to be a feasible approach for that issue. The immersed boundary (IB) method using the direct forcing approach [1, 2, 3] is one of the Cartesian grid methods widely used in incompressible flow simulations. In this method, velocities in the cells adjacent to the immersed boundaries are calculated by interpolations using the velocities at the boundaries and surrounding fluid cells, which may cause a reduced accuracy especially under low grid resolutions. Therefore, in this study, a new numerical scheme of a Cartesian grid method using a direct discretization approach is proposed for simulations of heat transfer and fluid flow, and the effectiveness of the proposed method is validated in some fundamental problems.

NUMERICAL METHOD

The governing equations consist of the incompressible Navier-Stokes, continuity and energy transport equations. The present method is based on the modified collocated grid system [4] in conjunction with the SMAC method. The Crank-Nicolson time discretization method is applied for the viscous and convection terms.

In the present method, the Navier-Stokes equation is discretized directly even in the boundary cells which are indicated in Fig. 1. Furthermore, in the discretized forms of the Navier-Stokes and pressure Poisson equations, the identical interpolation formulae both for fluxes and pressure gradients at the cell faces nearest to the boundaries are employed. These discretization procedures in the boundary cells ensure the momentum conservation, maintain the consistency of those discretized governing equations, and achieve higher prediction accuracy in the vicinity of boundaries accordingly [5].

Likewise, the basic idea of this direct discretization approach is extended to the temperature field [6]. The energy transport equation is discretized directly in the boundary cells involving either the Dirichlet (isothermal) or the Neumann (iso-heat-flux / adiabatic) boundary conditions in order to ensure the energy conservation in those cells. The temperature gradients in both the normal and tangential directions at boundaries are required in the present method for representing the Neumann boundary condition on the Cartesian grids which do not necessarily coincide with the body geometries. The tangential components of the temperature gradients at boundaries are calculated by the extrapolations from the surrounding temperature field. This treatment enables proper enforcement of the Neumann boundary condition and it is easy to extend to the three-dimensional problems.

RESULTS

The validity of the present method in the velocity field is assessed in a flow between concentric cylindrical walls as shown in Fig. 2. A constant pressure gradient in the circumferential direction is applied in the flow field and the no-slip boundary conditions are applied at the inner and outer walls. In addition to the present method, the voxel method where boundaries are expressed as stepwise geometries, and two kinds of the direct forcing IB methods [2, 3] which will be referred to as Case 1 and 2 respectively, are applied to compare the results. The Reynolds number based on the friction velocity and the channel half height $\delta$ is set to 2, and the Prandtl number is set to 0.7. Simulation has been conducted using the uniformly spaced grids in $x$-$y$ direction with five different grid spacings of $\Delta = 0.48 \sim 0.025\delta$, and the accuracy of each method is evaluated by comparing the result with the analytical solution. The relation between the $L^2$ norm error in the circumferential velocity and the grid spacing (Fig. 3) shows that the velocity in the voxel method converges at a rate of 1 against the grid spacing, while the results by the IB method and the present method converge at a rate of 2, meaning that these methods have second order accuracies for the velocity field. With regard to the convergence for the wall shear stress, the errors do not decrease in the voxel method and also in the IB method (Case 1) where an inconsistent scheme is employed for the discretizations of...
the Navier-Stokes and pressure Poisson equations in the boundary cells. In the IB method (Case 2) with a consistent scheme, a first order convergence rate is confirmed, whereas in the present method, nearly a second order convergence rate is established even for the wall shear stress and their error magnitudes for all the resolutions are considerably decreased compared to those by the other methods.

In order to investigate the accuracy for the temperature field, we have also conducted a simulation in a convective heat transfer problem on the same flow field under the several different types of thermal boundary conditions applied at the inner and outer walls. In this validation, two kinds of conventional methods are employed: the voxel method and the direct forcing approach (Case A) where the IB method of Case 2 is used for the velocity field and the similar approach based on the interpolation procedure is used for the temperature field. For the case of applying the isothermal boundary conditions both at the inner and outer walls \((T_i = T_o = 0)\), the errors in the temperature field (Fig. 4) show almost the same tendencies as those in the velocity field, namely, the temperature error converges at a rate of 1 against the grid spacing in the voxel method and a rate of 2 in the other two methods. As for the error in the wall temperature gradient, it does not converge in the voxel method, first and second order convergence rates are obtained in Case A and the present method respectively, and their error magnitudes in the present method are decreased compared to those in the other methods, suggesting the advantage in the accuracy of the present method.

The present method is also verified under the temperature Neumann boundary condition for the concentric cylindrical channel (Fig. 2). The Dirichlet boundary condition of the circumferentially-distributed temperature \((T_i = T_0 \sin \theta)\) is applied at the inner walls, and the adiabatic boundary condition \((\partial T / \partial r)_o = 0\) is applied at the outer wall. Because the temperature distribution imposed at the inner wall strongly affects the outer wall temperature, there exist a large temperature gradient in the tangential direction as can be seen in Fig. 5 (a). This temperature gradient at the Neumann boundary makes the prediction of the correct temperature field difficult by the Cartesian grid methods. The results are compared with those obtained by the independently-conducted simulation using a body-fitted-curved (BFC) grid with a sufficiently fine grid resolution. Comparing with the BFC results, it is clearly shown that the present method achieves a more precise prediction of the temperature distribution near the outer wall than the other two conventional methods (Fig. 5 (b)), and a second order convergence rate for the temperature is observed in the present method (Fig. 5 (c)). These results imply that, in the present method, the temperature gradients are correctly considered at the boundary cells where the Neumann boundary
Figure 4: $L^2$ error norms for the temperature field plotted against spatial resolution under the isothermal boundary conditions at the inner and outer walls.

Figure 5: Temperature distributions and the deviation of the temperature from the BFC result under the condition of the circumferentially-distributed temperature at the inner wall and the adiabatic boundary condition at the outer wall.

REFERENCES


